Pontryagin's principle for optimal control problems under uncertainties

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In this talk, we present a class of control problems with uncertainties. Let T be a given finite horizon, and let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space. Consider an ensemble of controlled state equations parametrized by the random variable $\omega \in \Omega$

$$\begin{cases} \dot{\mathbf{x}}_\omega(t) = A\mathbf{x}_\omega(t) + B(\omega)\mathbf{u}(t) + f(t,\omega) & \text{for a.e. } t \in [0,T], \\ \mathbf{x}_\omega(0) = x_0, & \\ \mathbf{u}(t) \in U, & \text{for a.e. } t \in [0,T], \end{cases} \tag{1}$$

where U is a closed metric space, x_0 is an initial data, A and $B(\omega)$ are linear operators in appropriate spaces (of finite or infinite dimension), and f is a smooth source term. An admissible control input $u : [0,T] \to H$ is a measurable function assumed to be ω-independent, which means that the *parametrized family of states* are driven by the same control. The optimal control problem is as follows

$$\text{Maximize } \{\mathbb{P}\big(\Psi(\mathbf{x}_\omega(T)) \le 0\big) \mid (\mathbf{x}_\omega, \mathbf{u}) \text{ satisfies } (1)\}$$

where $\Psi : H \to \mathbb{R}$ is a given function. The cost function evaluates the probability that the ensemble of controlled states verify a constraint at the final time.

In this talk, we will discuss the optimality conditions of the probust control problem and show that these conditions can be expressed as a Pontryagin principle.

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